

# Deuteron formation in nuclear matter

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## Abstract

We investigate deuteron formation in nuclear matter at finite temperatures within a systematic quantum statistical approach. We consider formation through three-body collisions relevant already at rather moderate densities because of the strong correlations. The three-body in-medium reaction rates driven by the break-up cross section are calculated using exact three-body equations (Alt-Grassberger-Sandhas type) that have been suitably modified to consistently include the energy shift and the Pauli blocking. Important quantities are the lifetime of deuteron fluctuations and the chemical relaxation time. We find that the respective times differ substantially while using in-medium or isolated cross sections. We expect implications for the description of heavy ion collisions in particular for the formation of light charged particles at low to intermediate energies.

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## I. INTRODUCTION

The formation of deuterons is the simplest multichannel reaction process in a heavy ion collision. Within a systematic quantum statistical approach the formation is driven by the Boltzmann collision integral that in turn requires the proper break-up cross sections. Besides electromagnetic disintegration/formation of the deuteron ( $np \rightleftharpoons \gamma d$ ) that is a generic two-body process, the treatment of three-body collision ( $NNN \rightleftharpoons Nd$ ) is more involved, and requires a solution of the three-body dynamics in medium. Collision rates are a central input into modern microscopic simulations of heavy ion collisions such as the Boltzmann-Uehling-Uhlenbeck (BUU) [1–3] or the quantum molecular dynamics approach (QMD) [4,5]. For the latest developments and recent applications to multifragmentation in heavy ion collisions see Ref. [6]. Danielewicz and Bertsch [2] started the numerical solution of coupled BUU equations including the deuteron formation. Later this model has been enlarged to include the production of three-particle clusters [7].

So far these approaches are fed with experimental cross sections, e.g.  $Nd \rightarrow NNN$  for the deuteron break-up, and the medium effect is taken into account by the Pauli-blocking of final states. However, in principle, as shown earlier those cross sections are changed substantially, when medium effects are consistently included [8,9]. Besides including these medium dependent cross sections in the respective simulations that is currently on its way, it is an interesting and important question to analyze the change of time scales induced by medium dependent reaction cross sections.

The correlated many-particle system is treated within the cluster mean field approach [10] that goes beyond the quasiparticle picture and in turn has been applied to a wide variety of many-particle physics problems. This approach is intimately related to the Dyson Equation Approach to Many-Body Green functions [11,12]. Both approaches lead to a systematic decoupling of the equation hierarchy and thus to effective few-body equations. For the two-particle case these are known as Feynman-Galitskii or Bethe-Goldstone equations. Recently, we have derived a corresponding equation for the three-body system for finite

temperatures [8,14] based on the formulation of Alt, Grassberger, and Sandhas (AGS) [13]. For *zero* temperatures three-body in-medium equations have been given in Ref. [12] for the related 2p-1h problem.

The respective one-, two-, and three-body equations are derived in an independent particle basis. That means we do not consider correlations in the surrounding matter that would eventually lead to a self-consistent treatment. However, such a treatment is technically involved and only very recently progress has been achieved towards this direction by including *two*-body correlations into the one-body spectral function [15]. Since presently our aim is different, i.e. to extract the formation and equilibration time scales, and the densities considered are rather low compared to the ones that require a self-consistent treatment, and since many technical problems still need to be tackled we presently do not consider a such an approach for the *three*-body case.

The life time and the relaxation time are determined by linearizing the Boltzmann equation or the corresponding rate equations. Therefore the respective integrals can be evaluated in thermal and chemical equilibrium. To this end we assume only small fluctuations, i.e. linear response of the system. This will be explained in the following section. The necessary three-body reaction input on the basis of Faddeev type equations will be given in Sect. III. The derivation of these equations is based on the Green function approach, e.g. [16]. In Sec. IV we will present numerical results and in the last section we summarize and give our conclusion.

## II. REACTIONS

The quantity of interest in the quantum statistical approach is the generalized quantum Boltzmann equation for the nucleon  $f_N(p, r)$ , deuteron  $f_d(P, R)$ , etc. Wigner distributions. For the time being we assume symmetric nuclear matter. These distributions are driven by a coupled system of Boltzmann equations, see, e.g. Ref. [2],

$$\partial_t f_N + \partial_p U \cdot \partial_r f_N - \partial_r U \cdot \partial_p f_N = \mathcal{K}_N^{\text{in}}[f_N, f_d] (1 - f_N) - \mathcal{K}_N^{\text{out}}[f_N, f_d] f_N,$$

$$\partial_t f_d + \partial_P U \cdot \partial_R f_d - \partial_R U \cdot \partial_P f_d = \mathcal{K}_d^{\text{in}}[f_N, f_d] (1 + f_d) - \mathcal{K}_d^{\text{out}}[f_N, f_d] f_d, \quad (1)$$

where  $U$  denotes the total mean field potential. The coupling between the different species is through the collision integrals  $\mathcal{K}[f_N, f_d]$  that are merely shown for the deuteron case in the following two equations relevant in the present context,

$$\begin{aligned} \mathcal{K}_d^{\text{in}}(P, t) = & \int d^3k \int d^3k_1 d^3k_2 |\langle kP|U|k_1k_2\rangle|_{dd \rightarrow dd}^2 f_d(k_1, t) f_d(k_2, t) \bar{f}_d(k, t) \\ & + \int d^3k \int d^3k_1 d^3k_2 |\langle kP|U|k_1k_2\rangle|_{Nd \rightarrow Nd}^2 f_N(k_1, t) f_d(k_2, t) \bar{f}_N(k, t) \\ & + \int d^3k \int d^3k_1 d^3k_2 d^3k_3 |\langle kP|U_0|k_1k_2k_3\rangle|_{pnN \rightarrow dN}^2 \\ & \quad \times f_N(k_1, t) f_N(k_2, t) f_N(k_3, t) \bar{f}_N(k, t) \\ & + \dots, \end{aligned} \quad (2)$$

where we used the abbreviations  $\bar{f}_N = (1 - f_N)$  and  $\bar{f}_d = (1 + f_d)$ , the ellipsis denote further possible contributions involving four and more body collisions (e.g.  $tp \rightleftharpoons dd$ ,  $hp \rightleftharpoons dd$ ) or processes like  $np \rightleftharpoons \gamma d$ , and  $\mathcal{K}_d^{\text{out}}(P, t)$  is given by

$$\begin{aligned} \mathcal{K}_d^{\text{out}}(P, t) = & \int d^3k \int d^3k_1 d^3k_2 d^3k_3 |\langle k_1k_2k_3|U_0|kP\rangle|_{dN \rightarrow pnN}^2 \\ & \quad \times \bar{f}_N(k_1, t) \bar{f}_N(k_2, t) \bar{f}_N(k_3, t) f_N(k, t) \\ & + \dots \end{aligned} \quad (3)$$

The quantity  $U_0$  appearing in Eqs. (2) and (3) is the in-medium break-up transition operator for the  $Nd \rightarrow NNN$  reaction and is calculated using the AGS equation as given in the next section. For the isolated three-body problem  $U_0$  determines the isolated break-up cross section  $\sigma_{\text{bu}}^0$  via [13,17]

$$\begin{aligned} \sigma_{\text{bu}}^0(E) = & \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1 d^3k_2 d^3k_3 |\langle kP|U_0|k_1k_2k_3\rangle|^2 \\ & \quad \times 2\pi \delta(E' - E) (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \end{aligned} \quad (4)$$

where  $|v_d - v_N|$  denotes the relative velocity of the incoming nucleon and deuteron.

So far the strategy has been to implement the *experimental* cross section into Eqs. (2) and (3). This has then been solved, for a specific heavy ion collision [2]. Using experimental cross

sections respectively isolated cross sections may not be sufficient in particular in the lower energy regime. The cross section itself depends on the medium, e.g. blocking of internal lines or self-energy corrections of the respective three-body Green functions. This has been shown in Refs. [8,9]. While the inclusion of the medium dependent deuteron break-up cross sections in a coupled BUU simulation of a heavy ion collision is a congruous application, it is instructive to study the influence of medium dependent cross sections in a relaxation time approximation, in addition. This would enable us to already arrive at conclusions on the possible effect for typical conditions at heavy ion collisions. After linearizing the Boltzmann equation it is possible to define a break-up time for small fluctuations of the deuteron distributions. To do so, we consider only the term given in Eq. (3) in the collision integral. For small fluctuations  $\delta f_d(t) = f_d(t) - f_d^0$  from the equilibrium distribution  $f_d^0$  linear response leads to [9]

$$\frac{\partial}{\partial t} \delta f_d(P, t) = -\frac{1}{\tau_{\text{bu}}(P, n_N)} \delta f_d(P, t), \quad (5)$$

where the “life time” of deuteron fluctuations that depends on the deuteron momentum  $P$  and the nuclear density  $n_N$  has been introduced earlier [9],

$$\begin{aligned} \tau_{\text{bu}}^{-1}(P, n_N) = & \frac{4}{3!} \int dk_N d^3k_1 d^3k_2 d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3 \rangle|^2 \\ & \times \bar{f}_N^0(k_1) \bar{f}_N^0(k_2) \bar{f}_N^0(k_3) f_N^0(k_N) 2\pi \delta(E - E_0), \end{aligned} \quad (6)$$

which can be related to the break-up cross section using Eq. (4).

Chemical equilibration results from break-up and formation reactions, according to the terms that contain the transition operator  $U_0$  in Eq. (2) and (3). The change of the total deuteron density is given by the integration of the deuteron momentum in Eq. (1). For a homogeneous system we can neglect the drift terms and only the reaction terms will change the total density, therefore we obtain the following rate equation

$$\frac{d}{dt} n_d(t) = -\alpha(t) n_N(t) n_d(t) + \beta(t) n_N^3(t), \quad (7)$$

where the introduced rate coefficient for disintegration  $\alpha$  is given by

$$\alpha(t) = \int d^3P d^3k \int d^3k_1 d^3k_2 d^3k_3 |\langle k_1 k_2 k_3 | U_0 | k P \rangle|_{dN \rightarrow pnN}^2 \times \bar{f}_N(k_1, t) \bar{f}_N(k_2, t) \bar{f}_N(k_3, t) \frac{f_N(k, t)}{n_N(t)} \frac{f_d(P, t)}{n_d(t)}, \quad (8)$$

and an analogous expression holds for the formation rate  $\beta$ . To obtain a time scale for the formation of chemical equilibrium, we consider a near-equilibrium situation.

In chemical equilibrium both rate coefficients are constant and related by detailed balance (time reversal invariance) resulting in

$$\beta = \frac{\alpha n_d^0}{(n_N^0)^2}. \quad (9)$$

Here we denote the equilibrium distribution functions by  $n_N^0$  and  $n_d^0$ . Assuming now small density fluctuations, viz.  $n_N = n_N^0 + \delta n_N$  and  $n_d = n_d^0 - \delta n_N/2$ , and after insertion into Eq. (1) a chemical relaxation time may be defined by

$$\frac{d}{dt} \delta n_d(t) = -\frac{1}{\tau_{\text{rel}}(n_N)} \delta n_d(t). \quad (10)$$

Using the definition of the rate coefficient  $\alpha$  given in Eq. (8) leads to following expression for the chemical relaxation time

$$\tau_{\text{rel}}^{-1}(n_N^0) = \int d^3k d^3P f_N^0(k) f_d^0(P) |v_d - v_N| \sigma_{\text{bu}}(k, P) \frac{n_N^0 + 4n_d^0}{n_N^0 n_d^0}, \quad (11)$$

The deuteron break-up cross section  $\sigma_{\text{bu}}$  is defined as in Eq. (4) but may now also include medium modifications of the transition operator  $U_0$  as will be explained in the next section. The deuteron equilibrium density  $n_d^0$  is related to the nucleon density via  $\mu_d = 2\mu_N$  for the chemical potentials.

### III. FINITE TEMPERATURE THREE-BODY EQUATIONS

The basis to derive effective in-medium few-body Green functions for finite temperatures is provided by the cluster mean field approximation. This approximation allows us to truncate the hierarchy of Green function equations in a controlled way. A general

introduction may be found in Ref [10]. This approach is related to the Dyson Equation Approach [11,12] extended here to finite temperatures [14,18]. It has been proven useful in the context of strongly coupled systems provided, e.g., in nuclear physics [12]. A calculable form is achieved by introducing ladder approximation. We consider generic elementary two-body interactions  $V_2$  only. The resulting equations for the decoupled one-, two- and three-body Green functions at finite temperatures (utilizing the Matsubara technique to treat finite temperatures [16]) will be given in the following. The *one-particle* Green function reads

$$G_1(z) = R_1^{(0)}(z) = (z - \varepsilon_1)^{-1}. \quad (12)$$

In mean field approximation the single quasiparticle energy  $\varepsilon_1$  is given by

$$\begin{aligned} \varepsilon_1 &= \frac{k_1^2}{2m_1} + \Sigma^{HF}(1), \\ \Sigma^{HF}(1) &= \sum_2 [V_2(12, 12) - V_2(12, 21)] f_2, \end{aligned} \quad (13)$$

and  $f_2 = f_N^0(\epsilon_2)$  the Fermi function<sup>1</sup>. This is given by

$$f(\omega) = \frac{1}{e^{\beta(\omega - \mu)} + 1}, \quad (14)$$

where  $\beta$  denotes the inverse temperature and  $\mu$  the chemical potential. The equation for the *two-particle* Green function  $G_2(z)$  reads

$$G_2(z) = N_2 R_2^{(0)}(z) + R_2^{(0)}(z) N_2 V_2 G_2(z), \quad (15)$$

Note that we use a matrix notation for the Fock space indices  $(1, 2, 3, \dots)$  for convenience.

The matrix for the two-body resolvent  $R_2^{(0)}(z)$  is then given by

$$R_2^{(0)}(12, 1'2'; z) = \frac{\delta_{11'} \delta_{22'}}{z - \varepsilon_1 - \varepsilon_2} \quad (16)$$

and the Pauli blocking factor  $N_2$  by

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<sup>1</sup>Here and in the rest of the paper we use equilibrium distribution functions only, and therefore we use  $f_N^0 \rightarrow f$  for notational simplicity.

$$\begin{aligned}
N_2(12, 1'2') &= \delta_{11'}\delta_{22'}(\bar{f}_1\bar{f}_2 - f_1f_2) \\
&= \delta_{11'}\delta_{22'}(1 - f_1 - f_2).
\end{aligned} \tag{17}$$

We use the abbreviation  $\bar{f} = 1 - f$ . The respective equation for the *three-particle* Green function relevant to describe three-body correlations in a medium is given by

$$G_3(z) = N_3 R_3^{(0)}(z) + R_3^{(0)}(z)[N_2 V_2] G_3(z), \tag{18}$$

where we have introduced the notation

$$[N_2 V_2](123, 1'2'3') = \sum_{\gamma=1}^3 (N_2 V_2)^{(\gamma)}(123, 1'2'3'), \tag{19}$$

$$(N_2 V_2)^{(3)}(123, 1'2'3') = (1 - f_1 - f_2) V_2(12, 1'2') \delta_{33'}. \tag{20}$$

The last equation is given for  $\gamma = 3$ . The channel notation is convenient to treat systems with more than two particles [17]. In the three-particle system usually the index of the spectator particle is used to characterize the channel. The Pauli factors  $N_3$  and the resolvents  $R_3^{(0)}$  read respectively

$$N_3(123, 1'2'3') = \delta_{11'}\delta_{22'}\delta_{33'}(\bar{f}_1\bar{f}_2\bar{f}_3 + f_1f_2f_3) \tag{21}$$

$$R_3^{(0)}(123, 1'2'3'; z) = \frac{\delta_{11'}\delta_{22'}\delta_{33'}}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3}. \tag{22}$$

Note that  $[N_2 V_2] \neq [N_2 V_2]^\dagger$  and  $N_3 R_3^{(0)} = R_3^{(0)} N_3$ . Although we assume a fermionic system the proper symmetrization of the equations is treated separately.

If the correlated pair, e.g. '12', and the spectator particle, e.g. '3', are uncorrelated in the channel (3) we may define a channel Green function  $G_3^{(3)}(z)$ . We generalize to the channel ( $\gamma$ ). The channel Green function is then defined by

$$G_3^{(\gamma)}(z) = \frac{1}{-i\beta} \sum_{\lambda} iG_2(\omega_{\lambda}) G_1(z - \omega_{\lambda}). \tag{23}$$

The summation is done over the bosonic Matsubara frequencies  $\omega_{\lambda}$ ,  $\lambda$  even,  $\omega_{\lambda} = \pi\lambda/(-i\beta) + 2\mu$ . The equation for the channel Green function is derived in the same way as for the total three-particle Green function given in Eqs. (18). The result is (no summation of  $\gamma$ )



$$G_3^{(\gamma)}(z) = N_3 R_3^{(0)}(z) + R_3^{(0)}(z)(N_2 V_2)^{(\gamma)} G_3^{(\gamma)}(z). \quad (24)$$

We are now ready to define the proper transition operator  $U_{\alpha\beta}$ ,

$$G_3(z) = \delta_{\alpha\beta} G_3^{(\alpha)}(z) + G_3^{(\alpha)}(z) U_{\alpha\beta}(z) G_3^{(\beta)}(z), \quad (25)$$

which in the zero density limit coincides with the usual definition of the transition operator with the correct reduction formula to calculate cross sections [19]. After proper three-body algebra we arrive at the following equation for the transition operator in medium, viz.

$$N_3 U_{\alpha\beta}(z) = (1 - \delta_{\alpha\beta}) [R_3^{(0)}(z)]^{-1} + \sum_{\gamma \neq \alpha} (N_2 V_2)^{(\gamma)} G_3^{(\gamma)}(z) U_{\gamma\beta}(z). \quad (26)$$

For  $N_3 \neq 0$  the equation may be multiplied by  $N_3^{-1}$  from the left. We now connect this equation to the two-body  $t$  matrix. To this end we define the following transition channel operator  $T_3^{(\gamma)}(z)$ ,

$$G_3^{(\gamma)}(z) = N_3 R_3^{(0)}(z) + R_3^{(0)}(z) N_2^{(\gamma)} T_3^{(\gamma)}(z) N_3 R_3^{(0)}(z). \quad (27)$$

The definition has been chosen so that it leads to the standard Bethe-Goldstone equation in the two-nucleon subchannel,  $T_3^{(3)}(z) = T_2(\tilde{z}) \delta_{33'}$ , where  $\tilde{z}$  is the frequency of the two-body subsystem embedded in the three-body system. Inserting eq. (27) into eq. (24) leads to the subchannel Bethe-Goldstone equation

$$T_3^{(\gamma)}(z) = V_2^{(\gamma)} + (V_2 N_2)^{(\gamma)} R_3^{(0)}(z) T_3^{(\gamma)}(z). \quad (28)$$

With the help of this equation it is possible to connect the AGS type equation directly to the two-body sub  $t$  matrix

$$N_3 U_{\alpha\beta}(z) = (1 - \delta_{\alpha\beta}) [R_3^{(0)}(z)]^{-1} + \sum_{\gamma \neq \alpha} N_2^{(\gamma)} T_3^{(\gamma)}(z) R_3^{(0)}(z) N_3 U_{\gamma\beta}(z). \quad (29)$$

Here we have written the Pauli factors occurring due to the surrounding matter explicitly. Note, that through eq. (28)  $T_3^{(\gamma)}(z)$  is also medium dependent.

Since  $N_3 > 0$  we may multiply Eq. (29) by  $N_3^{-1/2}$  from the left and by  $N_3^{1/2}$  from the right, and introduce  $U_{\gamma\beta}^* = N_3^{1/2} U_{\gamma\beta} N_3^{1/2}$ . Then

$$U_{\alpha\beta}^*(z) = (1 - \delta_{\alpha\beta})[R_3^{(0)}(z)]^{-1} + \sum_{\gamma \neq \alpha} N_3^{-1/2} N_2^{(\gamma)} T_3^{(\gamma)}(z) N_3^{1/2} R_3^{(0)}(z) U_{\gamma\beta}^*(z). \quad (30)$$

We may now introduce the transition channel operator  $T_3^{*(\gamma)} = N_3^{-1/2} N_2^{(\gamma)} T_3^{(\gamma)} N_3^{1/2}$  so that the equation looks formally as for the isolated case. The respective channel  $t$  matrix equation then is

$$T_3^{*(\gamma)} = V_3^{*(\gamma)} + V_3^{*(\gamma)} R_3^{(0)} T_3^{*(\gamma)}. \quad (31)$$

The explicit form of the effective potential arising in this equation reads (e.g. in the (3) channel)

$$\begin{aligned} V_3^{*(3)}(123, 1'2'3') &= N_3^{-1/2}(123) N_2^{(3)}(123) V_3^{(3)}(123, 1'2'3') N_3^{1/2}(1'2'3') \\ &= (1 - f_1 - f_2)^{1/2} (1 - f_3 + g(\varepsilon_1 + \varepsilon_2))^{-1/2} \\ &\quad \times V_2(12, 1'2') \delta_{33'} (1 - f_3 + g(\varepsilon_{1'} + \varepsilon_{2'}))^{1/2} (1 - f_{1'} - f_{2'})^{1/2} \end{aligned} \quad (32)$$

$$\simeq (1 - f_1 - f_2)^{1/2} V_2(12, 1'2') (1 - f_{1'} - f_{2'})^{1/2}. \quad (33)$$

The last equation holds for  $f^2 \ll f$  only and  $g(\omega) = [e^{\beta(\omega-2\mu)} - 1]^{-1}$ . Utilizing this approximation eq. (30) has been solved numerically using a separable potential for the strong nucleon-nucleon interaction [8].

#### IV. RESULTS

Since the Green functions have been evaluated in an independent particle basis the one-, two-, and three-particle Green functions are decoupled in hierarchy, as given in Eqs. (12), (15), (18). To solve the in-medium problem up to three-particle clusters the one-, two- and three-particle problems are consistently solved. This leads to the single particle self-energy shift Eq. (13), the two-body input including the proper Pauli blocking Eq. (31), and eventually to the three-body scattering state.

For technical reasons we have used some reasonable approximations valid at smaller effective densities. The nucleon self-energy has been calculated via Eq. (13), but in the three-body code we have approximated the self-energy by effective masses, i.e.

$$\varepsilon_1 = \frac{k_1^2}{2m} + \Sigma^{HF}(k_1) \simeq \frac{k_1^2}{2m^*} + \Sigma^{HF}(0). \quad (34)$$

For simplicity and speed of the calculation, we use angle averaged Fermi functions  $\langle\langle N_2 \rangle\rangle$  and  $\langle\langle N_3 \rangle\rangle$ , e.g.

$$\langle\langle N_3 \rangle\rangle = \frac{1}{(4\pi)^2} \int d\cos\theta_q d\cos\theta_p d\phi_q d\phi_p N_3(p, q, P_{\text{c.m.}}), \quad (35)$$

where the angles are taken with respect to  $P_{\text{c.m.}}$  and  $p, q$  are the standard Jacobi coordinates [17].

We have solved the three-body equation neglecting terms of the order  $f^2$ , using eq. (33) as the driving kernel in the AGS equations, which is sufficient, since the calculation is done below the Mott density of the deuteron.

A rank-one Yamaguchi potential [20] has been used in the calculations of the deuteron break-up cross section that includes the coupled  $^3S_1$ - $^3D_1$  and the  $^1S_0$  channels only. The parameters are given in Table I. To get an impression of the quality of the calculation the isolated cross section is given in Fig. 1 along with the experimental data on neutron deuteron scattering [21].

Unlike the isolated three-body problem the integral equation depends on the relative momentum of the three-nucleon system with respect to the medium, since the Fermi functions depend explicitly on the center of mass momentum of the three-body system. As a consequence one has to solve the three-body problem at a finite center of mass momentum  $P_{\text{c.m.}} = k_1 + k_2 + k_3$  in a medium that may be considered at rest  $P_{\text{med}} = 0$ . However, technically it is more convenient to let the three-nucleon system rest  $P_{\text{c.m.}} = 0$  and the surrounding medium move with  $P_{\text{med}} = -(k_1 + k_2 + k_3)$ . This procedure results in the least change of the three-body algebra and is possible since the dependence on  $P_{\text{c.m.}} - P_{\text{med}}$  is only through the Pauli blocking factors thus parametric.

For small momenta  $P_{\text{c.m.}}$  and small relative energies of the  $dN$ -system the influence of the medium is most pronounced. In Fig. 2 we see that the in-medium cross section is significantly *enhanced* compared to the isolated one. So it is not a justifiable approximation to include

medium effects only by the Pauli-blocking factors of the final states (see eq. (3)). That would effectively decrease the break-up cross section and might lead to wrong conclusions. The dominant effect is the weakening of deuteron binding energy (Mott-effect). This provokes also a shift of the threshold to smaller energies with raising densities. We observe that for higher energies the medium dependence of the cross section becomes much weaker, which *a posteriori* justifies the use of isolated cross sections (along with the impulse approximation) when higher energies are considered [2]. The dependence of the cross section on the center-of-mass momentum  $P_{\text{c.m.}}$  is shown in Fig. 3 for a fixed density and temperature. With the definition of a medium dependent cross section according to Eq. (4) we can rewrite for the deuteron break-up time Eq. (6)

$$\tau_{\text{bu}}^{-1}(P) = 4 \int dk_N |v_d - v_N| \sigma_{\text{bu}}(k, P) f(k_N). \quad (36)$$

To get an impression how much the medium dependence of the cross section shortens the deuteron break-up time we show in Fig. 4 the result of the above Eq. (36) with use of the medium cross section  $\sigma_{\text{bu}}^*$  calculated by solving Eq. (30) compared to the isolated  $\sigma_{\text{bu}}^0$ . For small deuteron momenta the break-up time is shorter by a factor of three at  $n = 0.007 \text{ fm}^{-3}$  and  $T = 10 \text{ MeV}$ .

We also calculate the chemical relaxation time given in Eq. (11). In Fig. 5 we see again that the medium dependence of the cross section shortens the time scale especially at higher densities.

## V. CONCLUSION AND OUTLOOK

Our results show that medium dependent cross sections in the respective collision integrals lead to *shorter reaction time scales*. Chemical processes become faster. This also effects the elastic rates that are related to thermal equilibration.

The basis of this result is the cluster Hartree-Fock approach that in our approximation includes correlations up to three particles in a consistent way. The equations driving the

correlations are rigorous. The respective one-, two- and three-body equations are solved, in particular for the three-particle case Faddeev/AGS type equations have been given in Ref. [8,9,14]. The AGS approach is particularly appealing since it allows generalizations to  $n$ -particle equations in a straight forward way. Results for the three-body bound state in medium will be published elsewhere [22]. As expected from the deuteron case, the triton binding energy changes with increasing density up to the Mott density, where  $E_t = 0$ .

The production rates, distribution of fragments, spectra etc. of light charged particles in heavy ion collisions at intermediate energies may change because of the much smaller time scales induced through the medium dependence compared to the use of free cross sections (respectively experimental cross sections).

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# FIGURES

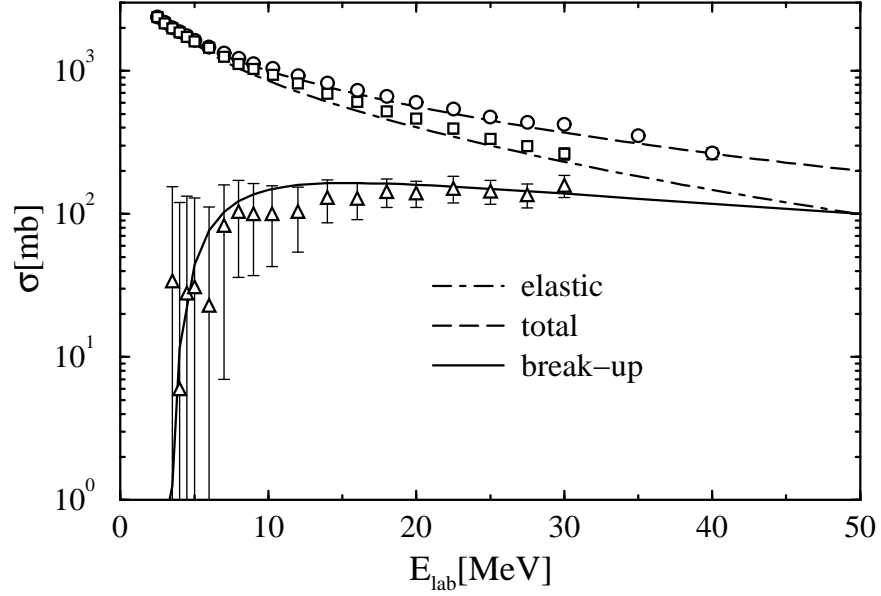


FIG. 1. A comparison of the total, elastic, and break-up cross sections  $nd \rightarrow nd$ ,  $nd \rightarrow nnp$  with the experimental data of Ref. [21].

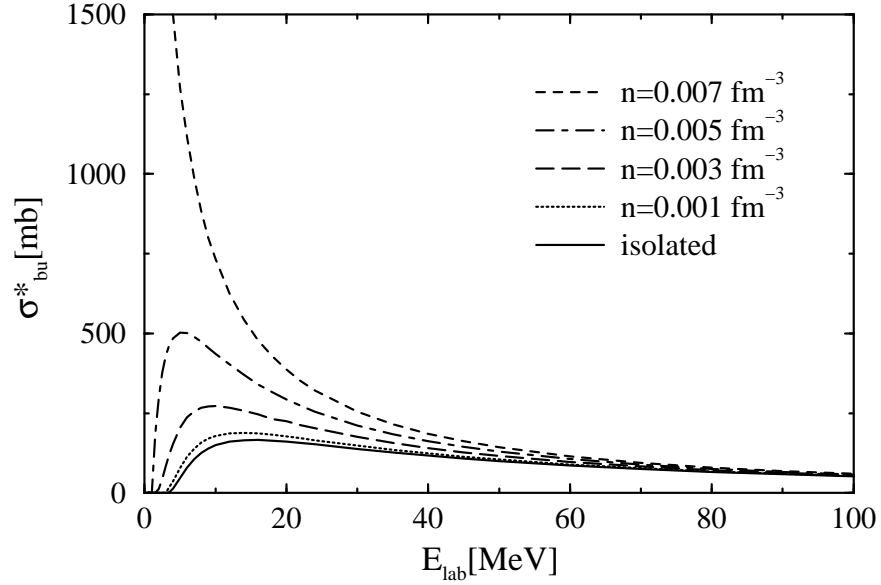


FIG. 2. In-medium break-up cross section at  $T = 10$  MeV and  $P_{c.m.} = 0$ . The isolated cross section is shown as a solid line, other lines show different nuclear densities as given in the legend.



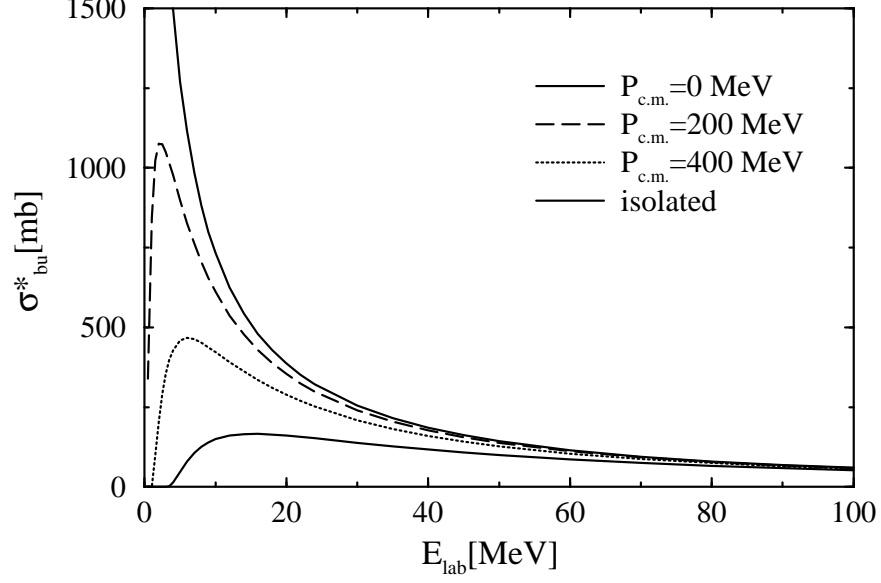


FIG. 3. In-medium break-up cross section at  $T = 10$  MeV and  $n = 0.007 \text{ fm}^{-3}$  for different center-of-mass momenta  $P_{\text{c.m.}}$ . The isolated cross section  $\sigma_{\text{bu}}^0$  is shown as solid line.

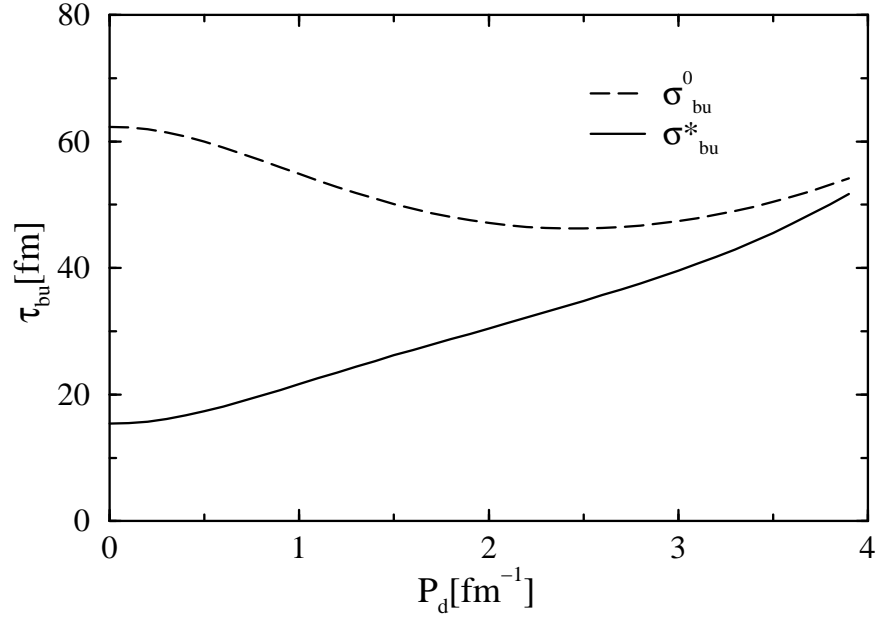


FIG. 4. Deuteron break-up time at  $T = 10$  MeV and nuclear density  $n = 0.007 \text{ fm}^{-3}$ . Solid line using the medium dependent cross section as given in Fig. 2, and dashed line using the isolated cross section.

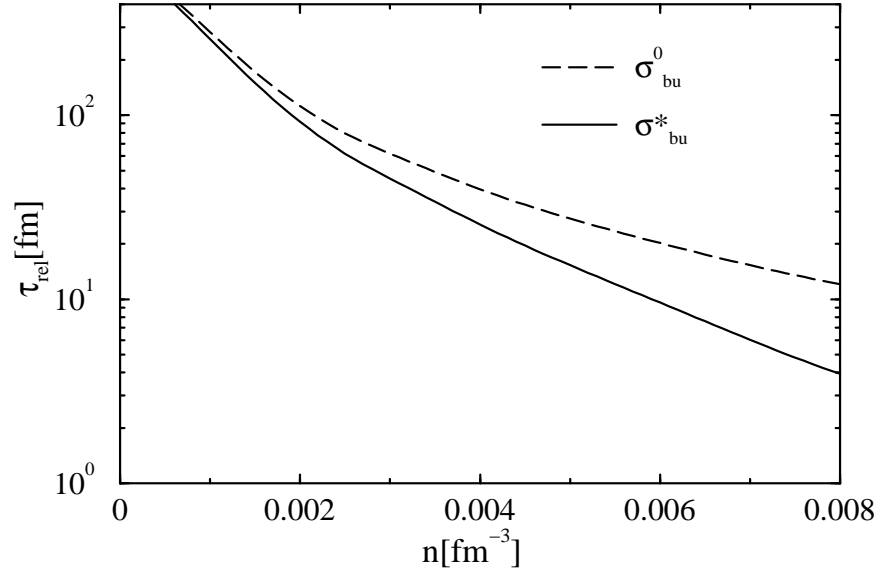


FIG. 5. Relaxation time for small fluctuations of the deuteron density from chemical equilibrium at a temperature of  $T = 10$  MeV as a function of the nucleon density. Line coding as in Fig. 4.

# TABLES

TABLE I. Parameters for the separable potential used to solve the three-body AGS equation. The form factors are  $g_{\kappa\ell}(p) = C_{\kappa\ell}p^\ell/(p^2 + \beta_{\kappa\ell}^2)^{\ell/2+1}$ . The binding energy of the isolated deuteron is  $E_d = 2.225$  MeV and of the isolated triton  $E_t = 7.937$  MeV [22].

$\kappa$	$C_0$	$C_2$	$\beta_0$	$\beta_1$
	[MeV fm <sup>-1</sup> ]		[fm <sup>-1</sup> ]	
<sup>3</sup> S <sub>1</sub> – <sup>3</sup> D <sub>1</sub>	8.63237	–38.8017	1.24128	1.94773
<sup>1</sup> S <sub>0</sub>	8.87026		1.17328	